Une approche rationnelle des milieux continus avec microstructure

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On the method of virtual power in Continuum Mechanics

JoMMS. 4: 281-292, 2009

Non-classical continua, pseudobalance, and the law of action and reaction

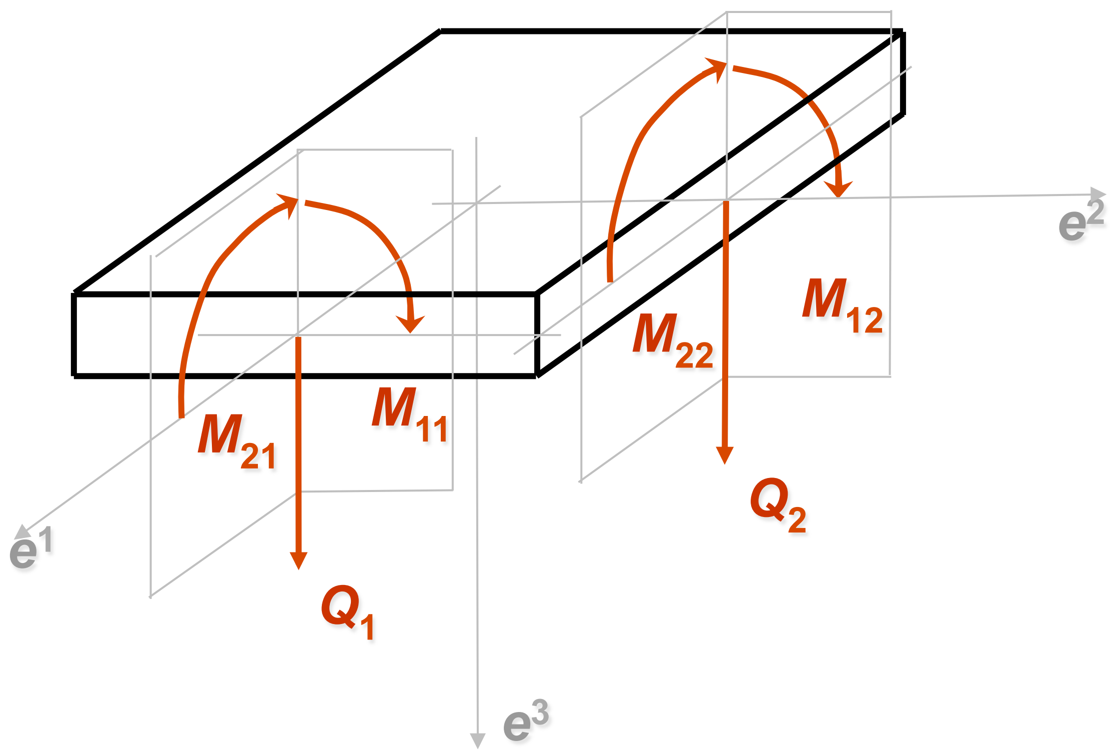
M&MOCS, forthcoming

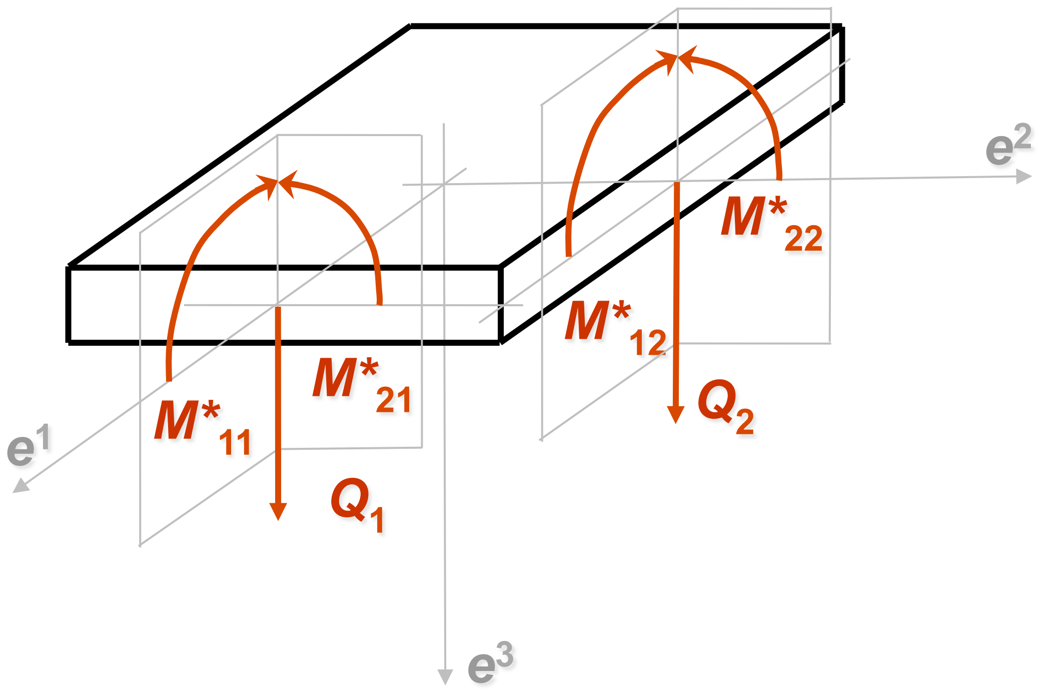
On the method of virtual power in the mechanics of non-classical continua

CISM Courses and Lecture Notes, forthcoming

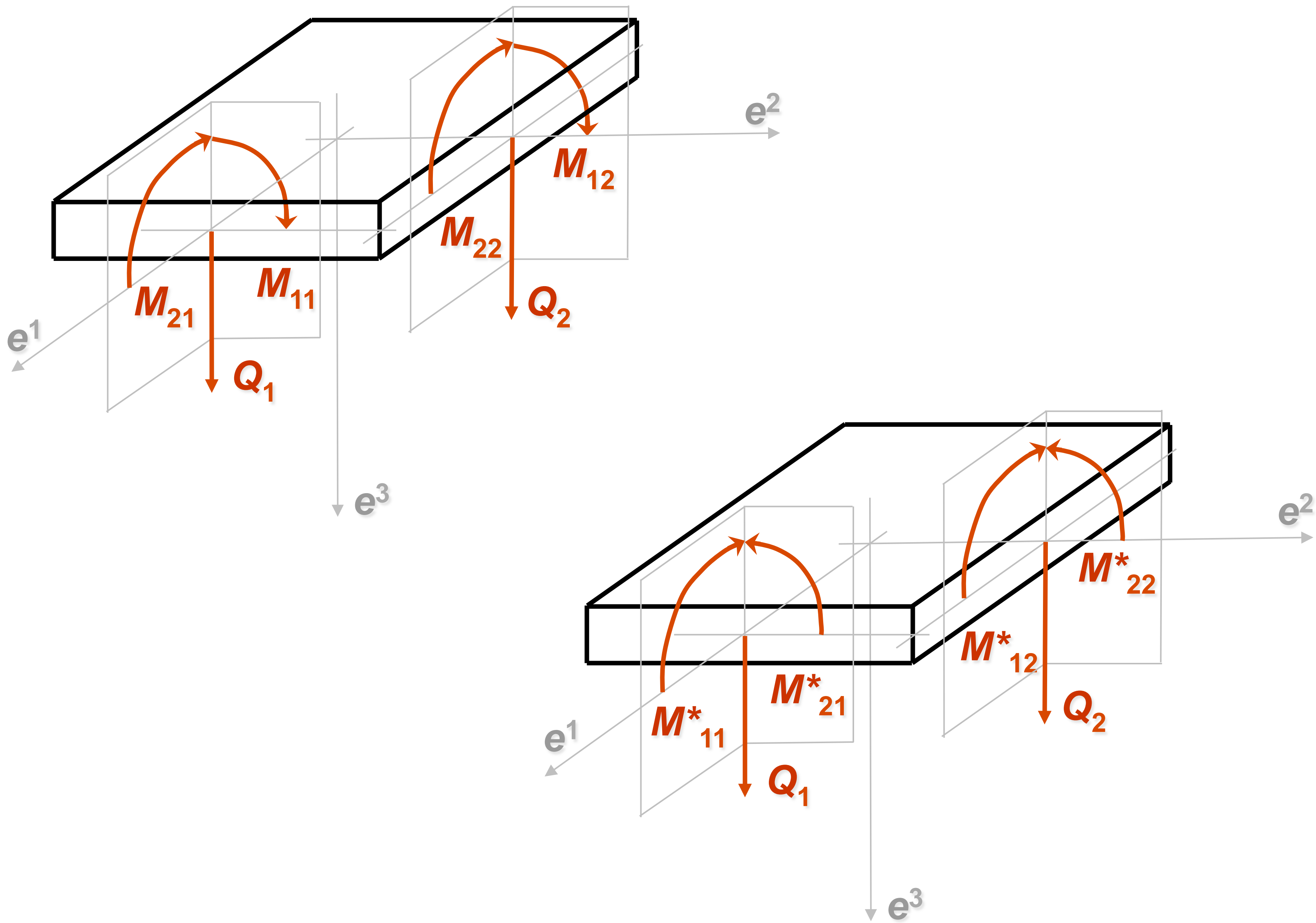
A rational approach to Cosserat continua with application to plate and beam theories

submitted


M21
M11
M22
M12
Q1
Q2
e1
e2
e3



e1
e2
e3
M*11
M*21
M*12
M*22
Q1
Q2



e2
M12
M22
M21
M11
Q2
e1
Q1
e3
e2
M*22
M*12
M*21
M*11
Q2
e1
Q1
e3

3D: Unconstrained Cosserat continuum

(3.19),(3.11)	$T^S_{ij,j} + e_{ikj}\, t_{k,j} + b_i = 0\,, \quad M_{ij,j} + c_i + 2\, t_i = 0$
(3.17)	$T^S_{ij} = \hat{T}^S_{ij}(\nabla^S v, 2\varphi, \nabla\omega)\,, \quad M_{ij} = \hat{M}_{ij}(\nabla^S v, 2\varphi, \nabla\omega)$ $t_i = \hat{t}_i(\nabla^S v, 2\varphi, \nabla\omega)$
(3.12)	$v_i = \hat{v}_i\,, \quad \omega_i = \hat{\omega}_i$
(3.23),(3.24)	$T^S_{\alpha n} + e_{\alpha\beta} t_\beta = s_\alpha\,,\ T^S_{nn} = s_n\,,\ M_{\alpha n} = m_\alpha\,,\ M_{nn} = m_n$

2D: Reissner plate

(5.5)	$Q_{\alpha,\alpha} + q = 0\,, \quad M_{\alpha\beta,\beta} + c_\alpha + e_{\alpha\beta} Q_\beta = 0$
(5.8)	$Q_\alpha = \hat{Q}_\alpha(\varphi, \nabla\omega), \quad M_{\alpha\beta} = \hat{M}_{\alpha\beta}(\varphi, \nabla\omega)$
(5.9)	$v_3 = \hat{v}_3 \;, \quad \omega_\alpha = \hat{\omega}_\alpha$
(5.10)	$Q_n = s_3\,, \quad M_{\alpha n} = m_\alpha$

1D: Timoshenko beam

(6.5)	$Q'_\alpha + q_\alpha = 0\,, \ \ M'_\alpha + c_\alpha - e_{\alpha\beta} Q_\beta = 0$
(6.8)	$Q_\alpha = \hat{Q}_\alpha(\varphi, \omega')\,, \ \ M_\alpha = \hat{M}_\alpha(\varphi, \omega')$
(6.9)	$v_3(l) = v_{3l}\,,\ v_3(0) = v_{30}\,, \ \ \omega_\alpha(l) = \omega_{\alpha l}\,,\ \omega_\alpha(0) = \omega_{\alpha 0}$
(6.10),(6.11)	$Q_\alpha(l) = P_{\alpha l}\,,\ Q_\alpha(0) = -P_{\alpha 0}\,, \ \ M_\alpha(l) = C_{\alpha l}\,,\ M_\alpha(0) = -C_{\alpha 0}$

3D: Constrained Cosserat continuum

(4.6)	$T^S_{ij,j} + b_i - \frac{1}{2}\, e_{ikj}\,(M_{kh,hj} + c_{k,j}) = 0$
(4.4)	$T^S_{ij} = \hat{T}^S_{ij}(\nabla^S v, \frac{1}{2}\mathrm{curl}\, v)\,, \quad M_{ij} = \hat{M}_{ij}(\nabla^S v, \frac{1}{2}\mathrm{curl}\, v)$
(4.11)	$v_\alpha = \hat{v}_\alpha\,, \quad v_n = \hat{v}_n\,, \quad v_{\alpha,n} = \hat{v}_{\alpha,n}$
(4.12),(4.13)	$T^S_{\alpha n} + \frac{1}{2}\, e_{\alpha\beta}(M_{nn,\beta} - M_{\beta h,h}) = s_\alpha + \frac{1}{2}\, e_{\alpha\beta}(c_\beta + m_{n,\beta})\,,$ $T^S_{nn} = s_n\,, \quad M_{\alpha n} = m_\alpha$

2D: Kirchhoff-Love plate

(5.19)	$M^{*S}_{\alpha\beta,\alpha\beta} + c^*_{\alpha,\alpha} - q = 0$
(5.17)	$M^{*S}_{\alpha\beta} = \hat{M}^{*S}_{\alpha\beta}(\nabla\nabla v_3)$
(5.22)	$v_3 = \hat{v}_3\,, \quad v_{3,n} = \hat{v}_{3,n}\,,$
(5.23)	$M^{*S}_{nn,n} + 2M^{*S}_{n\tau,\tau} = m^*_{\tau,\tau} - s_3 - c^*_n\,, \quad M^{*S}_{nn} = m^*_n$

1D: Euler-Bernoulli beam

(6.17)	$M''_\alpha + c'_\alpha + e_{\alpha\beta} q_\beta = 0$
(6.15)	$M_\alpha = \hat{M}_\alpha(\kappa)$
(6.20)	$v_\alpha(l) = v_{\alpha l}\,, \quad v_\alpha(0) = v_{\alpha 0}\,, \quad v'_\alpha(l) = v'_{\alpha l}\,, \quad v'_\alpha(0) = v'_{\alpha 0}$
(6.18),(6.19)	$M'_\alpha(l) + c_\alpha(l) = e_{\alpha\beta} P_{\beta l}\,, \quad M'_\alpha(0) + c_\alpha(0) = -e_{\alpha\beta} P_{\beta 0}\,,$ $M_\alpha(l) = C_{\alpha l}\,, \quad M_\alpha(0) = -C_{\alpha 0}$

THE END